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EFFECT OF CONSTITUTIVE MODELLING ON THE DYNAMIC DEVELOPMENT OF SHEAR BANDS IN VISCOPLASTIC MATERIALS*

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ABSTRACT. We model the viscoplastic response of a HY-100 steel by a Power law, and flow rules proposed by Litonski, Bodner and Partom, and Johnson and Cook. Each of these flow rules is first calibrated by using the torsional test data at a strain-rate of 3,300 sec^{-1}. These material models are then used to study the thermomechanical deformations of a block made of the HY-100 steel and undergoing simple shearing deformations at a nominal strain-rate of 5000 sec^{-1}. A material defect is simulated by assuming a non-uniform initial temperature distribution within the block. Whereas all of the flow rules used predict a rapid drop of the shear stress as a shear band forms, only for the Litonski Law for nonpolar materials, does an unloading elastic wave emanate outwards from the shear band.

INTRODUCTION. Noting that Batra (1987) has briefly reviewed the work done on shear bands till 1986, we discuss below some of the work done since then. For strain-rate hardening but thermally softening materials Wright and Walter (1987) found that the shear stress within a band collapses rapidly as the band grows. Batra and Kim (1989a) accounted also for material elasticity and work hardening effects and found that if the rate of collapse of the shear stress is large, then an unloading elastic wave emanates outwards from the shear band and propagates towards the boundaries of the specimen. The development of shear bands in plane strain problems have been studied, among others, by Anand et al. (1988), Needleman (1989), LeMonds and Needleman (1986a,1986b), Batra and Liu (1989a,1989b). These works have employed different flow rules and have modeled a material defect by introducing either a temperature perturbation or assuming the existence of a weak material at the site of the defect. Batra and Kim (1989b) have recently studied the development of a shear band in a block of HY-100 steel undergoing overall simple shearing adiabatic deformations and compared computed results with the experimental observations of Marchand and Duffy (1988). They found that the dipolar theory due to Wright and Batra (1987) and Batra (1987, 1989) and the Bodner-Partom (1975) law predict most of the features of the shear band.

We note that Molinari and Clifton (1987), Tzavaras (1987) and Wright (1989) have studied the problem analytically. For rigid/perfectly plastic materials, Wright (1989) has developed a criterion that ranks materials according to their tendency to form adiabatic shear bands. Hartley et al. (1987), Giovanola (1987), and Marchand and Duffy (1988) have reported the observed histories of the temperature and strain within a band as it develops.

Here we presume that the torsional experiments on thin-walled steel tubes can be analyzed by studying the thermomechanical deformations of a viscoplas-

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tic block undergoing overall adiabatic simple shearing deformations. We find the values of the material parameters appearing in different flow rules by solving an initial-boundary-value problem and comparing computed results with the experimental stress-strain curve at a nominal strain-rate of 3,300 sec^{-1}. These flow rules are then used to compute the initiation and growth of a shear band when the applied nominal strain-rate is 5,000 sec^{-1}. It is found that the rate of stress drop during the growth of a shear band as predicted by the Bodner-Partom law and the dipolar theory due to Wright and Batra (1987) is similar to that observed experimentally.

GOVERNING EQUATIONS. In terms of non-dimensional variables, equations governing the thermomechanical deformations of a viscoplastic block undergoing overall adiabatic deformations are (e.g. see Batra and Kim (1989a))

$$\rho\dot{v} = (s - \ell\sigma_{,y})_{,y} \qquad 0< y < 1, \tag{2.1}$$

$$\dot{\theta} = k\,\theta_{,yy} + s\,\dot{\gamma}_p + \ell\sigma\dot{d}_p, \qquad 0 < y < 1, \tag{2.2}$$

$$\dot{s} = \mu(v_{,y} - \dot{\gamma}_p), \tag{2.3}$$

$$\dot{\sigma} = \mu\ell(v_{,yy} - \dot{d}_p), \tag{2.4}$$

$$\dot{\gamma}_p = g(s,\sigma,\gamma_p,d_p,\theta,\ell), \tag{2.5}$$

$$\dot{d}_p = \ell h(s,\sigma,\gamma_p,d_p,\theta,\ell). \tag{2.6}$$

These equations, written for dipolar materials, reduce to those for non-polar materials when ℓ is set equal to zero. Here ρ is the mass density, v the velocity of a material particle in the direction of shearing, a superimposed dot indicates the material time derivative, s is the shearing stress, ℓ a material characteristic length, σ the dipolar stress, and a comma followed by y signifies partial differentiation with respect to y. Furthermore, k is the thermal conductivity, γ_p the plastic strain-rate, d_p the dipolar plastic strain-rate, μ the shear modulus, and θ is the temperature change from that in the reference configuration. Equation (2.1) expresses the balance of linear momentum and (2.2) the balance of internal energy, equations (2.3)-(2.6) are constitutive relations. The different viscoplastic flow rules differ in the functional forms of g and h and are given below in the next section.

For the initial conditions we take

$$v(y,0) = 0, s(y,0) = 0, \sigma(y,0) = 0,\ \theta(y,0) = \epsilon(1-y^2)^9\, e^{-5\,y^2}\ . \tag{2.7}$$

That is, in the initial rest state of the block, it is taken to be stress free. The initial temperature distribution simulates the defect or inhomogeneity in the block assumed to be present near the point y = 0 and the value of ϵ represents the strength of the defect.

We presume that the overall deformations of the block are adiabatic and the lower surface is at rest while the upper surface is assigned a velocity that increases linearly from 0 to 1 in time t_r and then stays at the constant value of 1.0. Thus,

$$\theta_{,y}(0,t) = 0,\ \theta_{,y}(1,t) = 0,\ v(0,t) = 0, \tag{2.8}$$

$$v(1,t) = t/t_r,\ 0 \le t \le t_r, \tag{2.9}$$

$$= 1, \; t \geq t_r,$$

and for dipolar materials, we also assume that

$$\sigma(0,t) = 0, \sigma(1,t) = 0. \tag{2.10}$$

Computations for the domain $-1 \leq y \leq 1$ and with boundary conditions $\sigma(-1,t) = 0, \sigma(1,t) = 0$ have given $\sigma(0,t) = 0$.

3. VISCOPLASTIC FLOW RULES. In order to calibrate the various flow rules against the shear stress-shear strain curve given by Marchand and Duffy (1988) for a strain-rate of 3,300 sec^{-1}, we solved numerically, the initial-boundary-value problem outlined above with

$$s(y,o) = 1.0, \gamma_p(y,o) = 0.012, \; v(y,o) = y, \theta(y,o) = 0^o\,c, \; \epsilon = 0,$$

$$t_r = 0.033, \; \rho = 7,860 \; kg/m^3, c = 473 \; J/kg^o c, \; k = 49.73 \; w/m^2 \; {}^o c, \; H = 2.5 \; mm,$$

$$\dot{\gamma}_o = 3,300 \; sec^{-1}.$$

Here H is the height of the block and $\dot{\gamma}_o$ is the average applied strain-rate. With no initial temperature perturbation, the block deforms uniformly and homogeneously and the dipolar effects vanish identically. As far as possible we kept the values of the strain-hardening exponent and the strain-rate-hardening exponent equal to those given by Marchand and Duffy (1988), and adjusted the values of other parameters till the computed stress-strain curve came out close to that given by Marchand and Duffy.

3.1 Litonski's Law for Nonpolar and Dipolar Materials. Wright and Batra (1987) generalized the constitutive relation proposed by Litonski (1977) to be applicable to nonpolar and dipolar materials. Batra and his co-workers (1987, 1988, 1989) have used it to study the initiation and growth of shear bands. It may be written as:

$$\dot{\gamma}_p = \Lambda s, \; \dot{d}_p = \frac{\Lambda}{\ell}\sigma, \tag{3.1}$$

$$\Lambda = \max\left[0, \left\{\frac{s_e}{(1-\alpha\theta)(1+\frac{\varphi}{\varphi_o})^n}\right\}^{1/m} - 1\}/bs_e\right], \tag{3.2}$$

$$s_e = (s^2 + \sigma^2)^{1/2}, \tag{3.3}$$

$$\dot{\varphi} = \Lambda s_e^2/(1 + \frac{\varphi}{\varphi_o})^n . \tag{3.4}$$

Here φ can be viewed as an internal variable that describes the work hardening of the material. Its evolution is given by equation (3.4). In equation (3.2), $(1-\alpha\theta)$ describes the softening of the material due to its heating, b and m characterize its strain-rate hardening, and φ_o and n its work hardening. The following values of material parameters resulted in a stress-strain curve that was close to the one observed experimentally.

$$\alpha = 0.00185/^{o}c, \ \varphi_o = 0.012, \ n = 0.107, \ m = 0.0117, \ b = 10^4 \text{ sec}, \ \ell = 0.005$$

3.2 Power Law. For nonpolar materials and assuming that there is no loading surface, this flow rule for the HY-100 steel can be written as

$$\dot{\gamma}_p = (10^{-4})\, s^{85.47} \left[\frac{\gamma}{0.012}\right]^{9.145} \left[\frac{\theta}{300}\right]^{-64.103} \tag{3.5}$$

Here θ is the current temperature in degrees Kelvin and γ is the total strain at a material particle.

3.3 Bodner-Partom Law. For the HY-100 steel, the constitutive relation proposed by Bodner and Partom (1975) can be written as

$$\dot{\gamma}_p = 10^8 \exp\left[-\frac{1}{2}\left(\frac{K^2}{3s^2}\right)\right]^n, \quad n = \frac{1200}{\theta}, \quad K = 1600 - 300\exp(-5W_p) \tag{3.6}$$

Here θ is the absolute temperature of a material particle and W_p is the plastic work done.

3.4. Johnson-Cook Law. The constitutive relation proposed by Johnson and Cook (1983) takes the following form for the HY-100 steel.

$$\dot{\gamma}_p = \exp\left[\left\{\frac{s}{(0.45 + 1.433\,\gamma_p^{0.107})(1-T^{0.7})} - 1.0\right\}/0.0277\right],$$

$$T = (\theta - \theta_o)/1200. \tag{3.7}$$

Here θ_o equals the ambient temperature.

4. DETERMINATION OF THE SIZE OF THE PERTURBATION. Here we model the cumulative effect of the change in the thickness of the specimen and possibly the slight variation in the material properties by assuming a nonuniform initial temperature distribution as given by Eqn. (2.7). For different flow laws, the value of ϵ was determined so as to initiate a shear band, as signified by a rapid drop in the shear stress, at a value of the average strain close to that found experimentally. The initial-boundary-value problem outlined in Section 2 with $t_r = 0.033$ was solved by the finite element method. Values of ϵ equal to 1^o c, 2^o c, 5^o c and 9^o c for the Litonski Law for nonpolar and dipolar materials, Power Law, and the Bodner-Partom Law and the Johnson-Cook Law, respectively, result in stress-strain curves shown in Fig. 1.

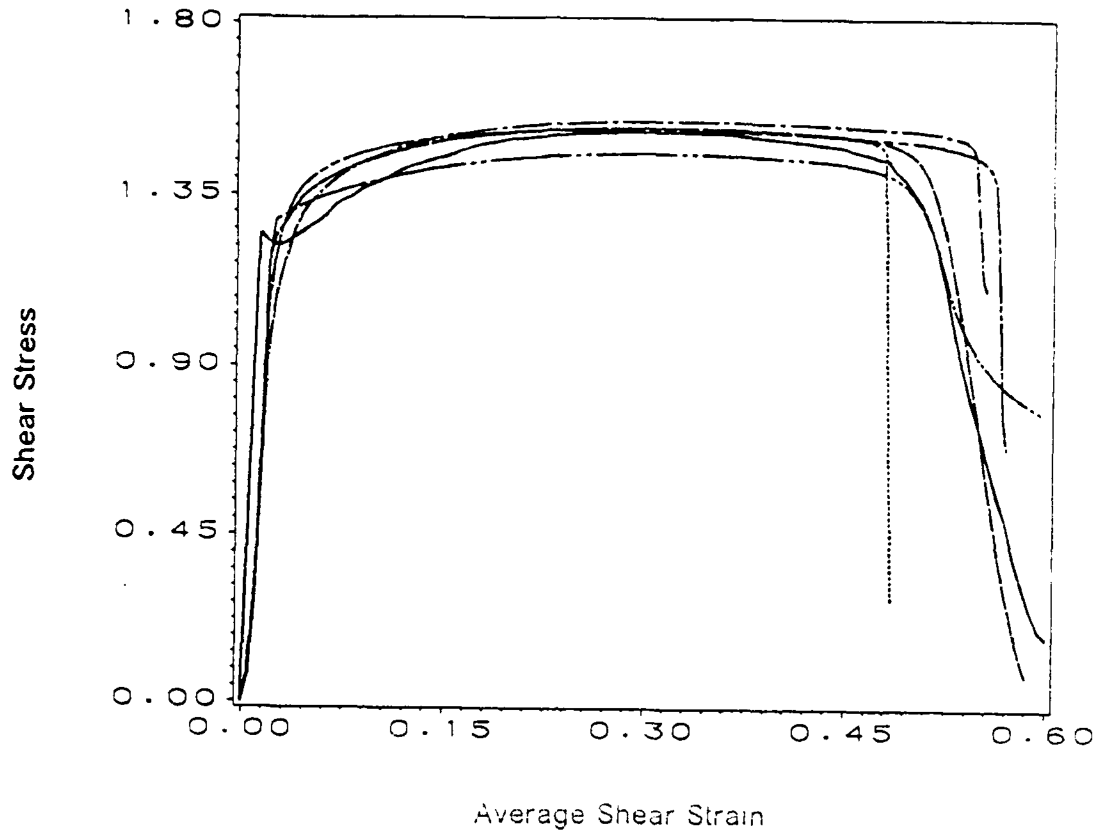

Fig. 1. Shear stress-shear strain curves computed with different flow rules and with different initial temperature perturbations.

_____experimental, ____..____..Bodner-Partom, ...Litonski (non-polar), __ __ __ Litonski (dipolar) ___ _ ___ _ ___ Power, ___ _ _ ___ _ _ Johnson-Cook.

These curves vividly reveal that until the time the shear stress begins to drop rapidly, all of the flow rules considered predict material behavior in reasonable agreement with the experimental observations. For nonpolar materials Litonski's Law, the Power Law and the Johnson-Cook Law give essentially a catastrophic drop in the shear stress with virtually no increase in the nominal shear strain. This does not agree with the experimental data since Marchand and Duffy observed that during the drop of the shear stress, the nominal strain increases by approximately 5 percent. The Litonski Law for dipolar materials and the Bodner-Partom Law for nonpolar materials do predict the gradual drop in the shear stress in agreement with the experimental data. However, for the Bodner-Partom Law the shear stress does not drop as much as it does during the tests and it reaches a plateau.

5. RESULTS FOR A NOMINAL STRAIN-RATE OF 5,000 SEC.$^{-1}$. With the values of material parameters and the size of the temperature perturbation found above kept fixed, we increased the prescribed velocity on the upper boundary so as to deform the block at a nominal strain-rate of 5,000 sec^{-1}. Note that the values of some of the non-dimensional variables appearing in the governing equations will change. For each one of the flow rules used, the shear stress attained a maximum value when the average shear strain was approximately equal to 0.30. For subsequent deformations, we have plotted in Figs. 2 and 3 the evolution of the shear stress and the particle velocity within the specimen. The value of the nominal shear strain at which the shear stress drops and the shear band initiates depends upon the flow rule used. However, in each case, the value of the nominal shear strain when a band initiates is noticeably more than the value at which the shear stress attains a maximum value.

For nonpolar materials, the rate of drop of the shear stress is highest for the Litonski law as compared to that for the other three flow rules used. For the Bodner-Partom law, the shear stress drops initially, but then seems to reach a plateau. For the Power law, the shear stress oscillates both in space and time and there was no unloading wave observed. With the Johnson-Cook law, the shear stress drops almost as rapidly as with the Litonski law, but seems to stay uniform throughout the specimen. For the Litonski law, as the shear stress drops, an unloading elastic wave emanates out of the shear band and travels towards the other end of the specimen. Batra and Kim (1989a) found this unloading wave and their computed wave speed was very close to the analytical value of $(\mu/\rho)^{1/2}$. The propagation of the wave is more clear from the particle velocity plot depicted in Fig. 3. We note that we assumed the existence of a yield surface only for the Litonski law. For other flow rules, plastic deformations are assumed to occur at all times.

For nonpolar materials, only Litonski's law as generalized by Wright and Batra was used. In this case, even though the shear stress drop was larger near the center as compared to that for nonpolar materials, no wave phenomenon was noticed. This becomes transparent from the velocity plot in Fig. 3.

For nonpolar materials, the velocity plots indicate that the particle velocity increases rapidly from zero at $y = 0$ to as high as 2 at a point close to $y = 0$ and then decreases to the prescribed value of 1 at $y = 1.0$. The overshoot in the particle velocity is highest for the Litonski law. The flow rule used affects the evolution of the particle velocity significantly. With the Johnson-Cook law, no oscillations in the particle velocity are observed. With the Bodner-Partom flow rule, no spatial oscillations in the particle velocity are seen but after a shear band has initiated, the velocity of a material particle oscillates in time. The spatial and temporal variation in the particle velocity with the Power law is noticeably different from that computed with the other three flow rules. A glance at the velocity and the shear stress plot seems to indicate that there is no unloading wave emanating out of the shear band in this case.

6. CONCLUSIONS. For overall adiabatic simple shearing thermomechanical deformations of a viscoplastic block, we first calibrated the four different flow rules so as to give essentially identical shear stress-shear strain curves at a nominal strain-rate of 3,300 sec^{-1}. Then, the size of the initial temperature perturbation was adjusted to yield the initiation of the shear band, as indicated by a significant drop in the shear stress for very little change in the nominal shear strain, at almost the same value of the nominal

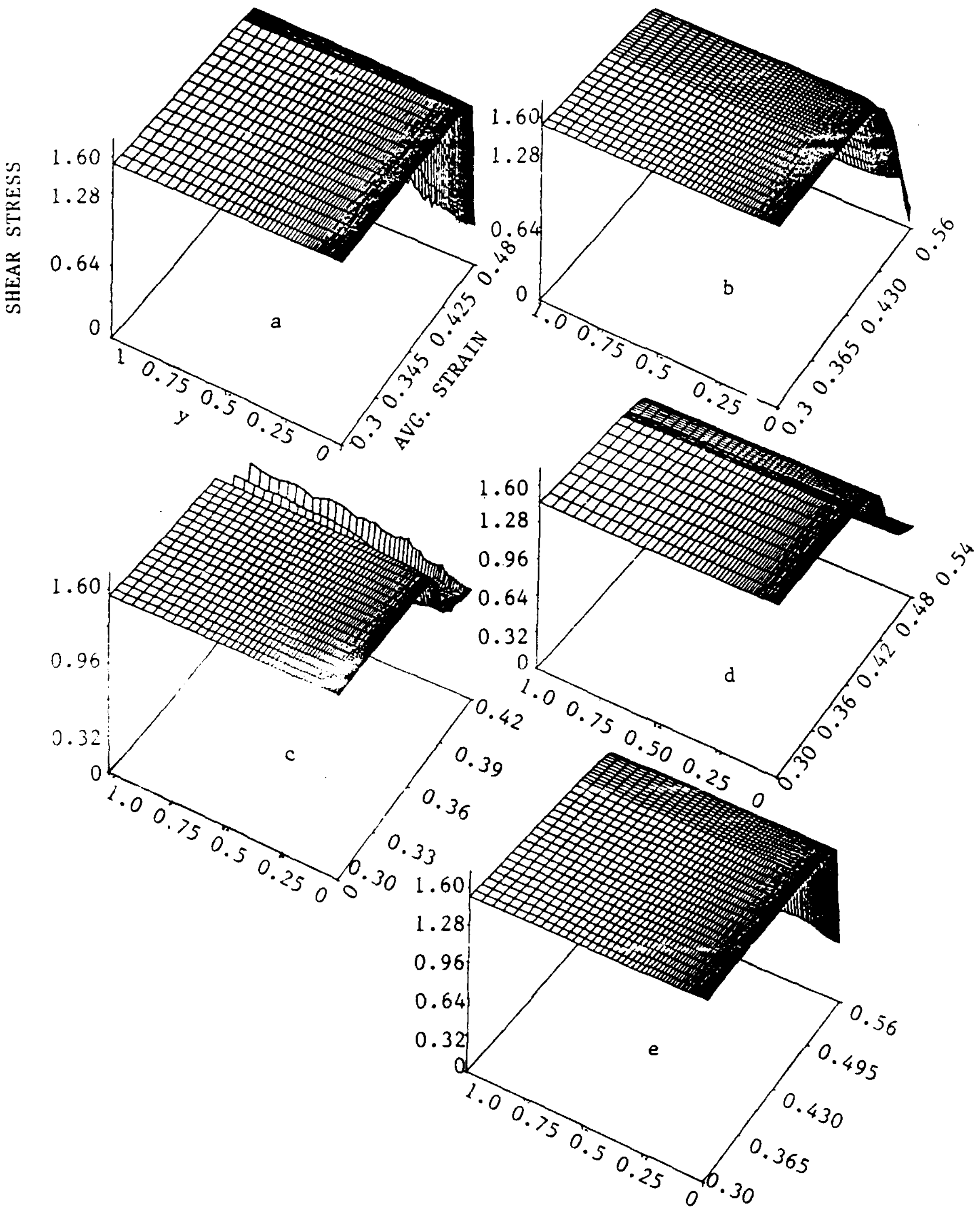

Fig. 2. The Evolution of the Shear Stress Within the Specimen After the Shear Stress has Attained Its Peak Value.

(a) Litonski Law (b) Litonski's Flow Rule for Dipolar Materials
(c) Power Law (d) Bodner-Partom Law, (e) Johnson-Cook Law

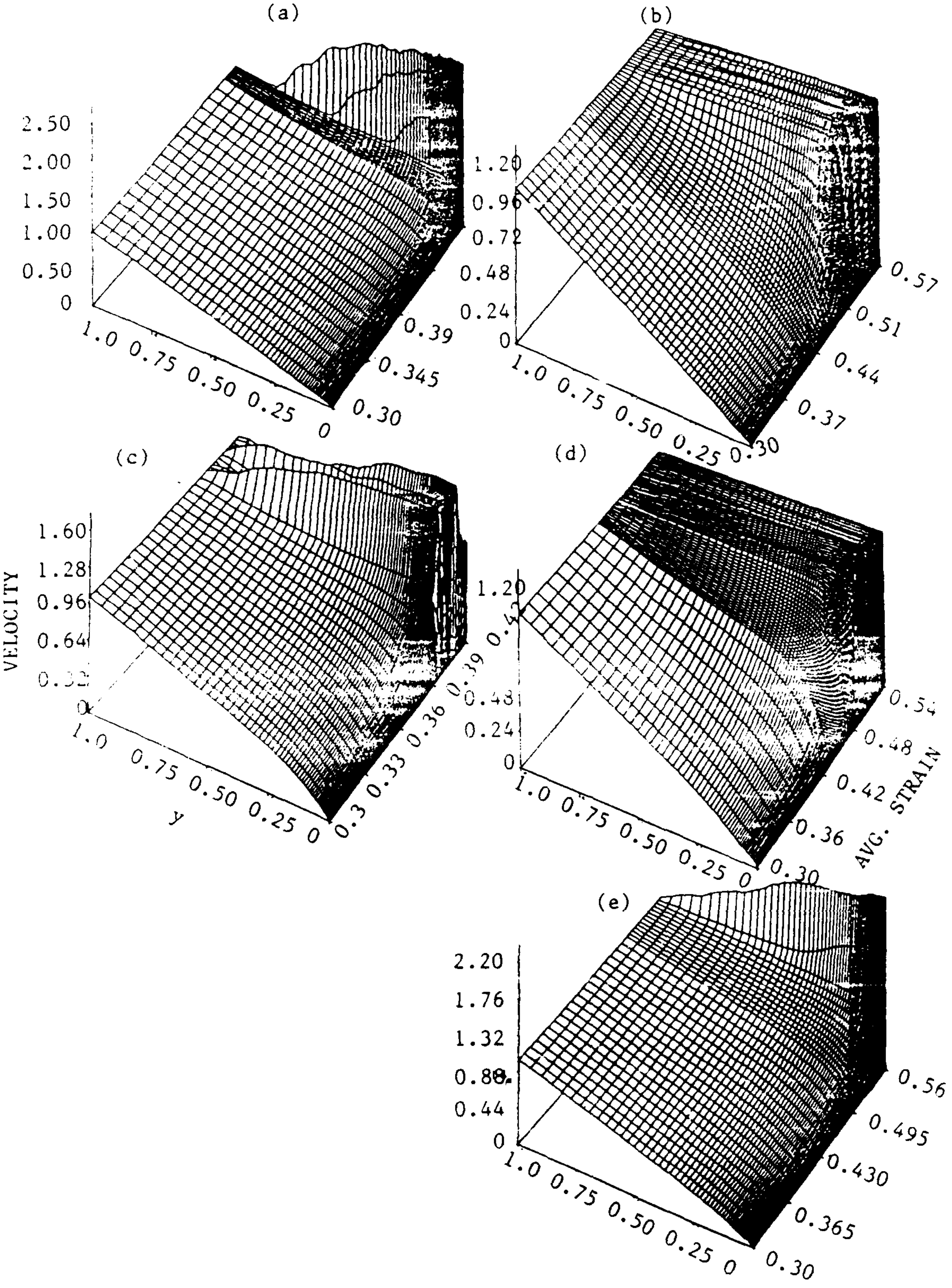

Fig. 3. The Evolution of the Velocity Field Within the Specimen After the Shear Stress has Attained Its Peak Value.

(a) Litonski's Law, (b) Litonski's Flow Rule for Dipolar Materials, (c) Power Law (d) Bodner-Partom Law, (e) Johnson-Cook Law strain. These flow rules when used to compute the initiation and growth of shear bands at a nominal strain-rate of 5,000 sec^{-1} gave noticeably different values of the nominal strain at which a shear band initiates. Also, the rate of drop of the shear stress as predicted by the Bodner-Partom law and the dipolar theory of Wright and Batra was closer to that observed experimentally. For nonpolar materials, the Litonski law predicts the emanation of an unloading elastic wave out of the shear band as it grows. The other three flow rules do give the overshoot in the particle velocity at the edges of the band as also given by the Litonski law, but do not predict the propagation of the unloading elastic wave. This could possibly be due to the use of a yield criterion for the Litonski law and not using any such criterion for the other flow rules.

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